③ $y'' + y' - 2y = 0$ Sol. $\boxed{y = 4e^{-2x} + 3e^{x}}$ ✓

$\frac{d\,e^{u}}{dx} = e^{u} \cdot \frac{du}{dx}$

$\frac{d\,e^{x^2}}{dx} = e^{x^2} \frac{d(x^2)}{dx}$

$= e^{x^2} \cdot (2x)$

La sol. la tengo que derivar.

$y' = 4(e^{-2x})(-2) + 3(e^{x})(1)$

⊛ $y' = -8e^{-2x} + 3e^{x}$

$y'' = (-8)(e^{-2x})(-2) + 3(e^{x})(1)$

⊛ $y'' = 16e^{-2x} + 3e^{x}$

sust. y y' y y'' en

$y'' + y' - 2y = 0$

$(\cancel{16e^{-2x}} + 3e^{x}) + (\cancel{-8e^{-2x}} + 3e^{x}) - 2(4e^{-2x} + 3e^{x}) = 0$

$\cancel{-8e^{-2x}} - 6e^{x}$

$0 = 0$

④ $y' = 25 + y^2$ Sol. $y = 5\tan 5x$ ↓ derivada

$y' = 5 \sec^2(5x)\,[5)$

$y' = 25\sec^2(5x)$

sust.

$25\sec^2(5x) = 25 + (5\tan 5x)^2$

$25\sec^2(5x) = 25 + 25\tan^2 5x$

$25\sec^2(5x) = 25(1 + \tan^2 5x)$

$25\sec^2(5x) = 25(\sec^2 5x)$

$\sec^2 x = 1 + \tan^2 x$

$y = 5\tan 5x$ si es solución.

UNIDAD (2) Ec. Dif. de 1er orden (1ª derivada)

$$y' + \frac{M}{N} = 0 \qquad \begin{matrix} M \\ N \end{matrix} (x, y)$$

$$\frac{dy}{dx} + \frac{M}{N} = 0 \Rightarrow M\,dx + N\,dy = 0$$

Método de separación de Variables

$$\int f(x)\,dx + \int g(y)\,dy$$

EJEMPLO.

$$5y\,dx + x\,dy = 0 \qquad \boxed{\cancel{\operatorname{sen}(xy)\,dy + x^2 y}\,dx = 0}$$

$$\frac{dx}{x} + \frac{dy}{5y} = 0$$

Aplicando la integral

$$\int \frac{dx}{x} + \int \frac{dy}{5y} = 0$$

$$\int \frac{dx}{x} + \int \frac{dy}{5y} = 0 \quad ; \quad \ln x + c + \frac{1}{5} \ln y + c = 0$$

$$\ln a^x = x \ln a$$

$$\ln y = -5 \ln x + C$$

$$\ln y = \ln x^{-5} + C$$ aplicando la exponencial

$$x^2 \cdot x^3 = x^{2+3} = x^5$$

$$e^{\ln y} = e^{\ln x^{-5} + c}$$

$$y = e^{\ln x^{-5}} \cdot e^c$$

$$y = x^{-5} \cdot C$$

$$\boxed{y = C x^{-5}}$$ general.

Método de ED Homogéneas.

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

No se pueden separar las variables.

① Cambiar una variable
$y = vx \qquad x = uy$

② Derivar los cambios
$dy = v\,dx + x\,dv \qquad dx = u\,dy + y\,du$

③ Sustituir el cambio y su derivada

④ Se procede a una separación de variables

⑤ Se regresa la variable original

Ejemplo Resolver $(4x-3y) + y'(2y-3x) = 0$

$(4x-3y) + \frac{dy}{dx}(2y-3x) = 0$

$y = vx$

$dy = v\,dx + x\,dv$

$\underbrace{(4x-3y)}_{M}\,dx + \underbrace{(2y-3x)}_{N}\,dy = 0$

multiplicando

Sust.

$[4x - 3(vx)]\,dx + [2(vx) - 3x][v\,dx + x\,dv] = 0$

$4x\,dx - 3vx\,dx + 2v^2x\,dx + 2vx^2\,dv - 3vx\,dx - 3x^2\,dv = 0$

Agrupar por diferenciales

$dx \qquad dv$

$(4x - 3vx + 2v^2x - 3vx)\,dx + (2vx^2 - 3x^2)\,dv = 0$

factorizar

$x \qquad x^2$

$x(4 - 6v + 2v^2)\,dx + x^2(2v-3)\,dv = 0$

$$\frac{x\,dx}{x^2} + \frac{2v-3}{4-6v+2v^2}\,dv = 0$$

$$\frac{x\,dx}{x^2} + \frac{2v-3}{4-6v+2v^2}\,dv = 0$$

$$\underbrace{\int \frac{dx}{x}}_{1} + \underbrace{\int \frac{2v-3}{4-6v+2v^2}\,dv}_{2} = 0$$

$$\int \frac{\frac{dw}{2}}{w} = \frac{1}{2}\int \frac{dw}{w}$$

② sust.

①

$$\int \frac{dx}{x} = \ln x + c$$

$y = vx$

$v = \frac{y}{x}$

②

$w = 4 - 6v + 2v^2$

$dw = (-6 + 4v)\,dv$

$dw = \cancel{4}\,2(\cancel{4}v - 3)\,dv$

$\frac{dw}{2} = (2v-3)\,dv$

$\frac{1}{2}\ln w + c = \frac{1}{2}\ln(4 - 6v + 2v^2) + c$

$\ln x + c + \frac{1}{2}\ln(4 - 6v + 2v^2) + c = 0$

$\ln x + \frac{1}{2}\ln\left(4 - 6\frac{y}{x} + 2\left(\frac{y}{x}\right)^2\right) = \underline{C}$

ED - Exactas

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

① Comprobar si es exacta.

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \quad ✓$$

$$\underbrace{\int M\,dx \qquad \int N\,dy}_{\text{sumar (No se repiten terminos)}}$$

2 estrategias

Integración ✓

Formula

IDI

Ejemplo $(5x + 4y)\,dx + (4x - 8y^3)\,dy = 0$

$\frac{M}{dy}$ $\qquad$ $\frac{N}{dx}$

④ $=$ ④ $\quad$ ✓ Exacta.

$\int (5x + 4y)\,dx = \frac{5x^2}{2} + 4xy$

$\int (4x - 8y^3)\,dy = 4xy - \frac{8y^4}{4}$

$y_g = \frac{5x^2}{2} + 4xy - 2y^4$

$y' + 2y = 2$ $\qquad \frac{dy}{dx} + 2y = 2$ $\qquad M\,dx + N\,dy = 0$

Factor integrante

$\frac{dy}{dx} + p(x)\,y = f(x)$ $\qquad p(x) = 2$

$\qquad f(x) = 2$

$\mu = e^{\int p(x)\,dx}$ $\qquad \int p(x)\,dx = \int 2\,dx = 2x$

$\mu = e^{2x}$

$$y = \frac{\int \mu f(x)\,dx}{\mu} + \frac{c}{\mu} = \frac{\int e^{2x}(2)\,dx}{e^{2x}} + \frac{c}{e^{2x}}$$

$$y = \frac{2e^{2x}(2)}{e^{2x}} + \frac{c}{e^{2x}} = 4 + \frac{c}{e^{2x}} = 4 + ce^{-2x}$$

$$\frac{2xy' - y = 3x^2}{2x}$$ $\qquad \frac{dy}{dx} + P(x)\,y = f(x)$ $\qquad \int x^n dx = \frac{x^{n+1}}{n+1}$

$y' - \frac{1}{2x}y = \frac{3x^2}{2x}$; $\quad y' - \frac{1}{2x}y = \frac{3}{2}x$ $\qquad n = \frac{1}{2}$

$P(x) = -\frac{1}{2x}$ $\qquad f(x) = \frac{3}{2}x$

$\mu = e^{\int P(x)\,dx}$ $\qquad \int P(x)\,dx = \int -\frac{1}{2x}\,dx = -\frac{1}{2}\ln x$

$= \ln x^{-1/2}$

$\mu = e^{\ln x^{-1/2}} = x^{-1/2}$

$$y = \frac{\int x^{-1/2}\left(\frac{3}{2}x\right)dx}{x^{-1/2}} + \frac{c}{x^{-1/2}} = \frac{\frac{3}{2}\int x^{1/2}\,dx}{x^{-1/2}} + cx^{1/2}$$

$$y = \frac{\frac{3}{2}\,\frac{x^{3/2}}{3/2}}{x^{-1/2}} + cx^{1/2} = \frac{x^{3/2}}{x^{-1/2}} + cx^{1/2} = \boxed{x^2 + c\sqrt{x}}$$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$y'' + y' + y = g(x)$$

$y'' + y' + y = 0$

Ec. caract. auxiliar

$y'' = r^2 \quad y' = r \quad y = \text{const.}$

$r^2 + r + c = 0$

aplicando formula Gral para ecs de 2° grado.

$r_1 = r_2$ reales

$y_g = c_1 e^{r_1 x} + c_2 x e^{r_2 x}$

$r_1 \neq r_2$ reales

$y_g = c_1 e^{r_1 x} + c_2 e^{r_2 x}$

r_1, r_2 complejas

$y_g = e^{\alpha x}[c_1 \cos \beta x + c_2 \operatorname{sen} \beta x]$

$\underset{\text{real}}{\alpha} + \underset{\text{imag.}}{\beta i}$

$r_1 = 2 - 4i$

$r_2 = 2 + 4i$

$y'' + y' + y = g(x)$

$g(x) =$ K, Polinomio, sen, cos, sen + cos, e

Coef. indeterminados

$y'' + y' + y = g(x)$

cualquier cosa

Variación de Parametros

1er Paso $y'' + y' + y = 0$

y_g

$g(x)$

y_p

$g(x)$

$y'' + 4y' + 4y = 8e^{-2x}$

1er Paso Hacer homogenea la ED.

$y'' + 4y' + 4y = 0$

2° Paso usar eqn. auxiliar

$r^2 + 4r + 4 = 0$

$r_1 = -2$

$r_2 = -2$

Sol. General

$y_g = c_1 e^{-2x} + c_2 x e^{-2x}$

3er Paso identificar $g(x)$

$g(x) = 8e^{-2x}$

Resolviendo por Coef. Indeterminados

4° Paso, se propone una nueva $g(x)$ que será la sol. particular.

$y_p = Ae^{-2x}$

$y_p = Axe^{-2x}$

$\boxed{y_p = Ax^2 e^{-2x}}$

La Sol. final $y_g + y_p$

$y = c_1 e^{-2x} + c_2 x e^{-2x} + Ax^2 e^{-2x}$

$$\sum_{n=1}^{\infty} a_n x^n = \square \qquad a_n + (n+3) = 1, 2, 3 \ldots$$

$$\sum_{n=1}^{\infty} (n-1) a_n x^n = \overset{(0)(a_1 x^1)}{(1-1) a_1 x^1} + (2-1) a_2 x^2 \ldots$$

adelantar

$$0 + \sum_{n=2}^{\infty} (n-1) a_n x^n$$

$$\sum_{n=1} x + \sum_{n=1} y = \sum (x+y)$$

$\Downarrow$

Fase (variable ficticia)

$2y'' - y' = 0$ empleando series de potencia.

Sol. $y = \sum_{n=0}^{\infty} a_n x^n$

$3x^2 \rightarrow (3)(2)x^{2-1}$

Derivando la solución

$$y' = \sum_{n=0}^{\infty} n a_n x^{n-1}$$

comprobar que el 1er termino NO sea cero

$$\boxed{y' = \sum_{n=1}^{\infty} n a_n x^{n-1}}$$

$$y'' = \sum_{n=1}^{\infty} n(n-1) a_n x^{n-2} = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

sust las derivadas en la ED

$$2\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=1}^{\infty} n a_n x^{n-1} = 0$$

comprobar que esten en fase con la potencia de "x"

$x^{2-2} = x^0$ $\qquad x^{1-1} = x^0$

Como estan en fase, se agruparan en una sola serie la $\sum$ debe empezar en el mismo número, empleando una variable ficticia.

$$2\sum_{n=2}^{\infty} n(n-1)\,a_n x^{n-2} - \sum_{n=1}^{\infty} n\,a_n x^{n-1} = 0$$

sust $n=2$: $K=n-2$, $K=0$; desp "n": $n=K+2$

$K=n-1$, $K=0$, $n=K+1$

$$2\sum_{K=0}^{\infty}(K+2)(K+2-1)\,a_{K+2}\,x^{K+2-2} - \sum_{K=0}^{\infty}(K+1)\,a_{K+1}\,x^{K+1-1} = 0$$

Agrupar

$$\sum_{K=0}^{\infty}\left[2(K+2)(K+1)a_{K+2}x^K - (K+1)a_{K+1}x^K\right] = 0$$

factorizar x^K

$$x^K\left[\underbrace{2(K+2)(K+1)a_{K+2} - (K+1)a_{K+1}}_{\text{desp la "a" mayor}}\right] = 0$$

$$a_{K+2} = \frac{(K+1)a_{K+1}}{2(K+2)(K+1)} = \frac{a_{K+1}}{2(K+2)}$$ Ecuación de recurrencia.

Transformada de Laplace.

$\int(\;) \longrightarrow$ formula $\qquad \int(+\;-) = \int(\,) - \int(\,) \rightarrow$ formula

$$\mathcal{L}\{f(t)+g(t)\} = \mathcal{L}\{f(t)\} + \mathcal{L}\{g(t)\}$$

$$\mathcal{L}\{C\} = \frac{C}{s}$$

$$\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\mathcal{L}\{\operatorname{sen}(wt)\} = \frac{w}{s^2+w^2}$$

$$\mathcal{L}\{\cos(wt)\} = \frac{s}{s^2+w^2}$$

$$\mathcal{L}\{\operatorname{senh}(bt)\} = \frac{b}{s^2-b^2}$$

$$\mathcal{L}\{\cosh(bt)\} = \frac{s}{s^2-b^2}$$

$$\mathcal{L}\{3t^2\} = 3\,\mathcal{L}\{t^2\} = 3\left(\frac{2!}{s^{2+1}}\right) = \frac{6}{s^3}$$

$$\mathcal{L}\{5\} = \frac{5}{s} \qquad \underset{a=-2}{\mathcal{L}\{e^{-2t}\}} = \frac{1}{s-a} = \frac{1}{s+2}$$

$$\mathcal{L}\left\{\begin{matrix} e^{at} \\ t^n \end{matrix} \cdot f(t)\right\} = \begin{cases} e^{at} & F(s-a) \quad \rightarrow \text{Traslación} \\ t^n & (-1)^n \dfrac{d^n}{ds^n} F(s) \quad \rightarrow \text{derivada} \end{cases}$$

$$\mathcal{L}\{e^{-2t} \cos t\}$$

$f(t) = \cos t$

$\mathcal{L} \downarrow$

$$\mathcal{L}\{\cos t\} = \frac{s}{s^2+1} = F(s)\Big|_{\substack{s-a \\ s+2}} = \boxed{\frac{s+2}{(s+2)^2+1}}$$

$a = -2$

$$\mathcal{L}\{t^2 \cos 2t\}$$

$f(t) = \cos 2t$ $\qquad (-1)^n \dfrac{d^n}{ds^n} F(s)$

$\mathcal{L} \downarrow$

$F(s) = \dfrac{s}{s^2+4}$ $\qquad n=2$, 2^a derivada

$$F'(s) = \frac{(s^2+4)(1) - (s)(2s)}{(s^2+4)^2} = \frac{s^2+4-2s^2}{(s^2+4)^2} = \frac{4-s^2}{(s^2+4)^2}$$

$$F''(s) = \frac{(s^2+4)^2(-2s) - (4-s^2)(2)(s^2+4)(2s)}{(s^2+4)^4}$$

$$= \frac{(-2s)(s^2+4)^2 - 4s(4-s^2)(s^2+4)}{(s^2+4)^4}$$

$$= \left[\frac{-2s}{(s^2+4)^2} - \frac{4s(4-s^2)}{(s^2+4)^3}\right](-1)^2$$